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Analysis of building parameter uncertainty in district heating for optimal control of network flexibility

Annelies Vandermeulen ^{1,2,4} , Ina De Jaeger ^{1,3,4*} , Tijs Van Oevelen ^{1,4} , Dirk Saelens ^{1,3}  and Lieve Helsens ^{1,2} 

¹ EnergyVille; Thor Park 8310, 3600 Genk, Belgium

² KU Leuven, Department of Mechanical Engineering; Celestijnenlaan 300, box 2421, 3001 Leuven, Belgium

³ KU Leuven, Department of Civil Engineering, Building Physics Section; Kasteelpark Arenberg 40, box 2447, 3001 Leuven, Belgium

⁴ VITO NV; Boeretang 200, 2400 Mol, Belgium

* Correspondence: ina.dejaeger@kuleuven.be

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Abstract: Network flexibility is the use of the thermal capacity of water contained in the district heating network pipes to store energy and shift the heat load in time. Through optimal control, this network flexibility can aid in applications such as peak shaving and operational heat pump optimisation. Yet, optimal control requires perfect predictions and complete knowledge of the system characteristics. In reality, this is not the case and uncertainties exist. To get an insight in the importance of these uncertainties, this paper studies the influence of imperfect knowledge of building parameters on the optimal network flexibility activation and its performance. It is found that for the optimisation of heat pump operation, building parameter uncertainties do not present large risks. For peak shaving, a more robust result can be achieved by activating more network flexibility than may be required.

Keywords: district heating, optimal control, heat demand flexibility, building parameter uncertainty, robust control

1. Introduction

To limit air pollution and green house gas emissions, a fundamental change in our energy system is required. In 2019, heating and cooling in the tertiary and residential sectors were responsible for 41.7 % of the total final energy use in the EU28 [1], while 79 % of energy used in European households went to space heating (SH) and domestic hot water (DHW) [2]. Furthermore, 75 % of the energy used for heating and cooling of buildings is based on fossil fuels, while only 18 % originates from renewable and residual energy sources (R²ES) (of which 90 % biomass) [3]. The heating and cooling sector for buildings thus represents a large fraction of the total energy use and is a viable opportunity to improve the system efficiency and the energy source portfolio.

Energy efficiency for the heating and cooling sector can be improved by district heating and cooling (DHC) systems in areas with a large heat/cold density, i.e. a large heat/cold demand per square kilometer. As Frederiksen and Werner stated, the fundamental idea of district heating (DH) is found in local synergies between heat sources and demand [4]. By connecting sources and demand through a pipe network, new heat and cold sources can be unlocked, such as combined heat and power (CHP), waste incineration, industrial residual heat, combustible renewables and geothermal sources, thereby improving energy efficiency and operational costs of the energy system. Connolly et al. [5] found that the inclusion of DHC in an EU energy efficiency strategy for 2050 can reduce the total costs for the heating and cooling of buildings by 15 %.

31 To increase the share of renewable and residual energy sources (R^2ES), their intermittency must be
32 dealt with. One possible solution is to introduce energy flexibility in the energy system. Its definition
33 is as follows [6]: ‘Energy flexibility is the ability to shift the energy injection into or energy extraction from
34 a system in time to bypass system limitations.’ By introducing energy flexibility, integration of R^2ES
35 can be improved by e.g. preventing curtailment. In this respect, DHC systems offer an interesting
36 opportunity; they contain multiple thermal energy storage systems (TES), such as water storage tanks,
37 aquifers, borefields, building thermal inertia, and the network itself. Intelligently deploying TES to
38 create energy flexibility [7] can pave the way to large shares of R^2ES .

39 Within this context, this paper focuses on energy flexibility created by the thermal capacity of the
40 water contained in DH network pipes, referred to as network flexibility from now on. By temporarily
41 increasing/decreasing the supply temperature in the DH network, the network is charged/discharged.
42 This way, energy can be stored for a while, bridging the gap between heat generation and heat demand.
43 A detailed description of a typical network flexibility activation can be found in [6].

44 By solving an optimal control problem (OCP), two applications of network flexibility are
45 considered in this paper. There is operational heat pump optimisation in which the interaction
46 with the day-ahead market is optimised, and peak shaving in which the use of an expensive and/or
47 polluting peak unit is minimised [8–10]. In the literature, other applications of network flexibility can
48 be found: CHP optimisation [11–13], R^2ES integration [14–16] and providing ancillary services [17].

49 However, the OCPs described in these studies all consider perfect predictions and perfect
50 knowledge of the system model and parameters. However, this is not the case in reality and the
51 OCP solution will deviate from the actual optimal control strategy. This study investigates the
52 influence of these deviations on the control performance. Before going into the novelty and the specific
53 research questions of this paper, the uncertainties that play a role in DH systems are introduced first,
54 followed by a discussion on robust control of energy systems with uncertainty: how to determine a
55 control strategy that can achieve a satisfactory result in (almost) all possible cases?

56 1.1. Uncertainties in district heating systems

57 Kim et al. [18] divided uncertainty into three categories. *Model uncertainties* are caused by a lack of
58 knowledge regarding the physical system and/or the necessity to simplify and neglect certain aspects
59 to keep the model solvable within acceptable time. *Process uncertainties* either refer to inaccurate
60 actuators and sensors, or to the inability to measure certain system states. *Forecast uncertainties* relate to
61 the imperfect forecasts made of system disturbances such as weather, electricity prices, R^2ES generation,
62 etc.

63 One example is heat demand uncertainty, which is in fact the result of other uncertainties.
64 The main contributors are: user behaviour predictions, weather forecasts and unknown building
65 construction. Of these, the former two are related to forecasts, while the latter belongs to the model
66 uncertainties category.

67 To accommodate the user behaviour uncertainties, several tools have been set up to stochastically
68 generate user behaviour profiles describing indoor temperature set-points, electrical appliance usage,
69 internal heat gains and DHW use. These are mostly based on surveys and hence represent the typical
70 behaviour of a certain population. Such tools include StROBe [19], Strathclyde University Demand
71 Profile Generator [20], DELORES [21] and a Japanese activity-based modelling tool [22].

72 Regarding weather predictions, often weather servers that provide regular weather forecasts can
73 be used to analyse weather uncertainties. For example, by combining imperfect weather forecasts with
74 the corresponding measurements of the weather as it actually occurred, Oldewurtel [23] analysed the
75 influence of these uncertainties for building heating.

76 The final contribution to heat demand uncertainty is the imperfect knowledge regarding building
77 construction and building energy performance. Especially on district or city level, building energy
78 performance related data is often unavailable [24,25]. Due to the lack of detailed input data on building
79 level, archetype buildings are often used. Archetype buildings are buildings that are considered to be

80 representative for a larger group of buildings. As an example, the TABULA project defines archetype
81 dwellings, i.e. typical dwellings, for multiple European countries [26]. For Belgium, 30 archetype
82 buildings are characterised in terms of their geometry and U-values for roof, ground floor, exterior wall
83 and windows. Thanks to the rising popularity and availability of geographical information systems
84 and geospatial data, the building geometry of all individual buildings can be included within district
85 energy simulations. However, thermal quality data of the building envelope is still rarely available,
86 although they are collected in some countries for the calculation and allocation of building energy
87 performance certificates. Unfortunately, these data are often not shared due to privacy issues. To
88 overcome this issue, De Jaeger et al. [bijna gepubliceerde paper Ina] developed a method to estimate
89 the thermal quality of the building envelope based on construction year and geometrical data of the
90 building based on statistical data from the Flemish energy performance certificates database.

91 In this paper, only the last source of heat demand uncertainty, the imperfect knowledge of building
92 parameters, is discussed.

93 1.2. Robust control of energy systems

94 With these uncertainties, the optimal solution of a deterministic OCP may be far from optimal
95 for the actual system, leading to a reduced performance. Hence, OCPs have been reformulated in
96 the literature to integrate uncertainties and to reduce the associated risk. Three approaches can be
97 discerned, ranging in complexity.

98 Firstly, deterministic model predictive control (MPC) is a first step towards improved robustness.
99 In short, an MPC solves an OCP with a receding horizon, i.e. at frequent points in time the OCP is
100 solved with updated forecasts and system state measurements. The resulting optimal control strategy
101 is then applied to the actual system [27]. Although the embedded OCP still does not consider the
102 uncertainties, the regular update of relevant predictions and states ensures that the MPC control
103 actions can adapt through time, all the while trying to minimise the objective. This technique is applied
104 by Arnold and Göran [28] who alleviated prediction errors of electricity demand and R²ES generation
105 in an electricity system with connected TES systems. They analysed the MPC performance by running
106 Monte-Carlo simulations and concluded that the TES systems provided the MPC with an opportunity
107 to deal with most of the prediction errors, thereby preventing unplanned start-ups of plants.

108 A second approach uses stochastic modelling to determine the robust optimal control of a system.
109 This is done by incorporating probability distributions for the stochastic parameters into the OCP.
110 Different types of stochastic modelling can be found. In single-stage stochastic programming, all
111 control actions are decided at one instance. This is e.g. the case in chance-constrained programming.
112 Here, the chance that a certain constraint will be violated is limited to a certain extent. Bruninx et al. [29]
113 applied such a chance constraint problem to ensure that the energy demand in an electricity system
114 would be successfully generated and delivered in e.g. 95% of the cases.

115 Two-stage optimisation problems are solved in two stages, as explained by Verrilli et al. [30]:
116 ‘In two-stage stochastic programs, the decision variables are divided into two groups: the first-stage variables,
117 which have to be decided before the actual realisation of the uncertain parameters becomes available, and the
118 second stage or recourse variables, which can be decided once the random events occur. These recourse variables
119 are also interpreted as correction actions to compensate any infeasibility from the first-stage decisions.’ This
120 technique has been applied multiple times. Wang et al. [31] applied it to the optimal control of a
121 building energy system. To test the robust optimal control problem, they compared it to an MPC
122 by running Monte-Carlo simulations of both controllers. They concluded the robust optimal control
123 and the MPC reached about the same performance, but the stochastic OCP could do so with a single
124 evaluation across the whole time horizon. Tian [17] optimised the operation of a CHP connected to
125 both the electricity system and a DH system with a two-stage stochastic problem. The goal was to
126 offer ancillary services and participate in the electricity spot market while the electricity demand is
127 uncertain. Interestingly, network flexibility is applied here to increase the CHP profits, yet no DH
128 system uncertainties were included.

129 Other stochastic programming techniques found in the literature include scenario robust
130 optimisation, in which a number of carefully selected scenarios are combined [18] in one OCP. Options
131 to select such scenarios include Sample Average Approximations [30], Monte-Carlo sampling [32],
132 Latin Hypercube Sampling [18] and the point-estimate strategy [33,34]. Monte-Carlo least squares
133 regression analysis [35] and min-max optimisations (worst-case optimisations) have also been applied
134 for robust control of energy systems [36,37].

135 Finally, a third approach to reach robust control is by combining the first two: MPC and stochastic
136 optimal control. While Oldewurtel [23] integrated a chance-constrained program for building energy
137 system control into an MPC, Rantzer [38] and Verrilli et al. [30] both developed an MPC containing a
138 two-stage optimisation problem for DH system control.

139 This overview shows that plenty of research in robust control of energy systems has been done.
140 However, to the authors' knowledge, there has been no research yet in robust control of network
141 flexibility with respect to heat demand uncertainty or any other form of uncertainty in the DH system
142 itself. Hence, the exploratory study presented in this paper focusing only on the impact of building
143 parameter uncertainty provides a valuable contribution to the scientific literature.

144 1.3. Novelty and research questions

145 The main novelty of this paper is the assessment of building parameter uncertainties, leading
146 to an uncertainty in the heat demand magnitude¹, impacting the network flexibility activation in
147 DH systems based on a deterministic OCP. Two applications of network flexibility are studied: 1)
148 operational heat pump optimisation in which the interaction of a central DH heat pump with the
149 day-ahead electricity market is studied, 2) peak shaving.

150 This study indicates how sensitive the optimal network flexibility activation is to the building
151 parameter uncertainties and hence a change in heat demand magnitude, and how much risk is
152 associated with adopting a control strategy based on wrongly estimated building parameters. This
153 paper shows whether simple measures can provide less risk and/or higher profits leading to a more
154 robust control strategy. It is a first step in estimating the importance of robust network flexibility control
155 and to the development of that robust control. The following research questions will be considered:

- 156 1. How does the optimal network flexibility activation (i.e. the control strategy) alter when the
157 building parameters are different?
- 158 2. How sensitive is the network flexibility performance to the applied control strategy (and hence
159 to uncertainty)?
- 160 3. Does this preliminary study lead to insights for a more robust activation of network flexibility?

161 In this paper, the considered case study is described first. Then, in Section 3, the methodology for
162 the optimal control, the uncertainty on the heat demand and the uncertainty analysis is introduced.
163 Subsequently, the results are presented in Section 4, followed by the discussion in Section 5. Finally,
164 the conclusions are formulated in Section 6.

165 2. Case study: *GenkNET*

166 The influence of building parameter uncertainty on network flexibility is tested by optimising the
167 control of *GenkNET*. This is a fictive DH system based on the city of Genk, Belgium. To set up this case
168 study, steps 1-4 in Figure 1 were followed. First, the geometrical data from 7775 buildings located in
169 Genk were collected from a CityGML LOD2 model. Then, Genk was divided into 9 neighbourhoods
170 and for every neighbourhood the average construction year was determined by a *Google Streetview*
171 scan. To determine the user behaviour of the people inhabiting these buildings, user behaviour profiles

¹ Uncertainty on the heat demand magnitude refers to a heat demand profile that has been scaled up/down with an uncertain (time-variable) factor. The 'magnitude' term is used in this text to emphasise that there are no timing changes. For an example of heat demand profiles that only have magnitude changes, please refer to Figure 5.

172 (temperature set-points, internal heat gains, DHW use, etc.) were generated with the stochastic toolbox
 173 StROBe [19].

GenkNET DATA COLLECTION	
1. Collect geometry from CityGML LOD2 model for all buildings	
2. Divide Genk into 9 residential neighbourhoods connected to the DH system	
3. Estimate average construction year per neighbourhood	
4. Allocate stochastic occupant behaviour based on StROBe	
OPTIMISE NEIGHBOURHOOD HEAT DEMAND PROFILES	CALCULATE UNCERTAINTY ON HEAT DEMAND PROFILES
5. Allocate building envelope parameters based on TABULA	9. Calculate 1 archetype per neighbourhood
6. Allocate stochastic occupant behaviour based on StROBe	10. Determine realistic input distributions for building envelope parameters per archetype based on probabilistic method
7. Optimise heat demand per building using <i>modesto</i>	11. Sample building envelope parameters for 500 variants per archetype
8. Sum heat demand profiles for all buildings in neighbourhood	12. Simulate heat demand of each variant using IDEAS
	13. Calculate CV(Q) per archetype based on LDC
14. Create 100 variants of heat demand profiles (step 8) using CV(Q) (step 13) per neighbourhood	
15. Create 100 variants of GenkNET heat demand profiles by random selection of each neighbourhood	

Figure 1. A flow chart describing the different steps taken to determine the *GenkNET* heat demand profiles including uncertainties (relevant to both Sections 2 and 3).

174 To limit the computational complexity of this case study and to prevent the simultaneous
 175 simulation of 7775 buildings, a thorough aggregation was carried out. Every neighbourhood in
 176 *GenkNET* is now represented by one substation that has to deliver the heat demand of the entire
 177 neighbourhood, neglecting the distribution network in a neighbourhood. For more details on this
 178 aggregation, please refer to [6]. This leads to the DH system layout shown in Figure 2. The nominal
 179 supply and return temperatures in this DH system are taken to be 57 °C and 37 °C, respectively. The
 180 pipe sizes are determined by the sizing procedure presented in [6].

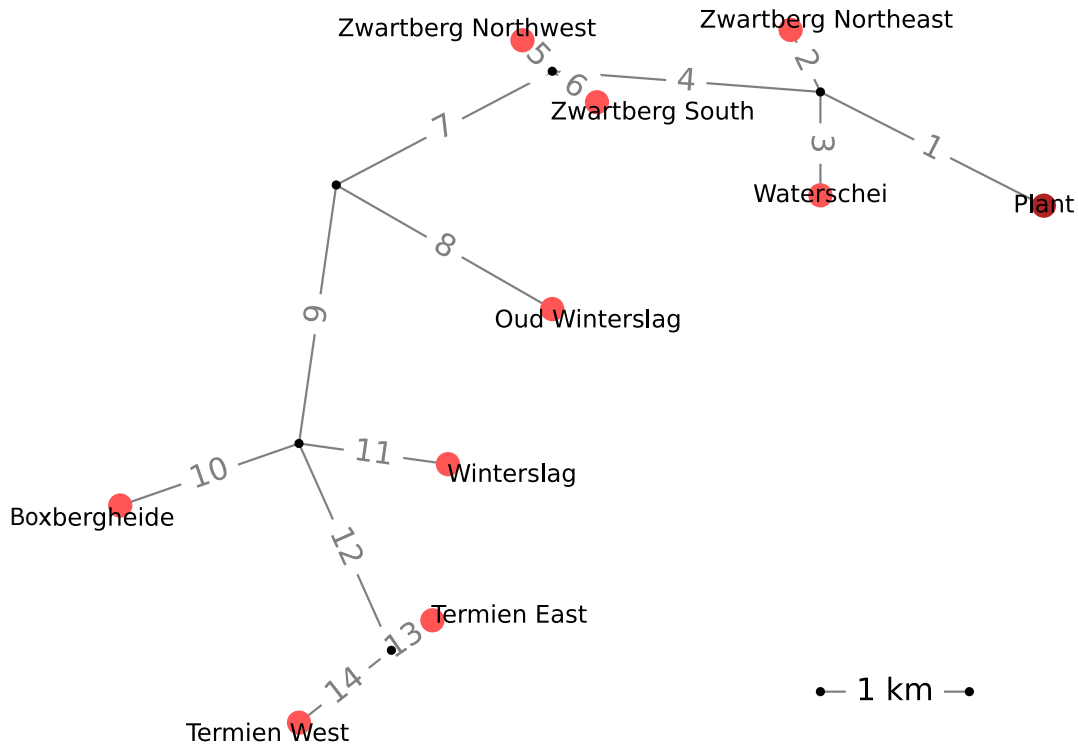


Figure 2. The lay-out of the aggregated *GenkNET*, indicating the position of the 9 neighbourhoods. There is a single heat generation site in the north-east of the network. The network pipes are indicated by the numbered lines.

181 Instead of a whole year analysis, only a limited number of days is tested in this paper. To
 182 select a representative set of days, two aspects are considered: 1) the overall heat demand, leading
 183 to a distinction between winter and transitional (spring and autumn) days. The summer days are
 184 not considered, as summer heat demands proved to be too low for interesting network flexibility
 185 activations and hence interesting results. 2) The day-ahead electricity price profile can be either stable
 186 and positive (small Δp_e), volatile and positive (large Δp_e), or become negative during the day (negative
 187 p_e). The electricity price p_e corresponds to the BELPEX day-ahead market prices in 2014. This leads to
 188 a selection of 6 days, given in Table 1. These days will be referred to as <season>_<electricity price>,
 189 according to the names of the columns and rows of Table 1.

Table 1. The nine days selected for the *GenkNET* case. These days will be referred to as <season>_<electricity price>.

Heat demand El. price	Winter	Trans(itional)
Small Δp_e	16/01	29/03
Large Δp_e	14/01	17/11
Neg(ative) p_e	16/02	16/03

190 In this paper, the heat generation unit is either a central air-to-water heat pump or a base/peak
 191 plant combination. The six selected days account for different electricity price profiles which allow
 192 to study different heat pump cases, as the operational heat pump optimisation heavily depends on
 193 the electricity price variation through time. To study the base/peak plant combination in more depth,
 194 different base load ratios are studied. The base load ratio r_b defines the capacity of the base unit $\dot{Q}_{b, \max}$
 195 relative to the peak heat demand of the analysed day $\dot{Q}_{\text{dem}, \max, \text{day}}$. Three base load ratios are tested:
 196 60, 80 and 95 %. Note that this will lead to base plant sizes that are different for every case (day and
 197 base load ratio).

$$r_b = \frac{\dot{Q}_{b, \max}}{\dot{Q}_{\text{dem}, \max, \text{day}}} \quad (1)$$

198 3. Methodology

199 This section presents the methodology used. Firstly, it discusses the optimal control problems
 200 that will be solved in this study. Secondly, the set-up of the heat demand profiles with uncertainty is
 201 presented. Finally, the methodology of the uncertainty analysis to assess the influence of the building
 202 parameter uncertainty on network flexibility is described.

203 3.1. Deterministic optimal control

204 To determine the optimal network flexibility activation, the toolbox *modesto* [39] is used. It
 205 contains a library of (non-linear) DH component models, including pipe, substation and heat generation
 206 models. The models as they are used in this study are presented in Appendix A. These models were
 207 developed specifically for determining optimal network flexibility activations. Hence, these models are
 208 suited to model the temporary network temperature changes in the DH network and the corresponding
 209 energy storage that take place during a network flexibility activation. For more information on the
 210 interactions that take place during a network flexibility activation, we refer to [6]. However, note that
 211 the models are completely deterministic and do not take into account any uncertainties.

212 *modesto* can automatically assemble the *GenkNET* DH system optimisation model based on
 213 its topology and a selection of models and optimisation objective. For each considered case, the
 214 network topology remains the same, yet the heat generation site and optimisation objective is changed
 215 depending on the studied case. In case of heat pump or peak shaving optimisation, either a heat
 216 pump model or a base/peak plant model is included. The optimisation objective depends on the heat

217 generation site, Equations 2 and 3 show the objectives C_{HP} and C_{PS} for the operational heat pump and
 218 peak shaving optimisation, respectively. In these objectives, \dot{W} is the electrical work done by the heat
 219 pump to generate the heat, $p_e(i)$ is the day-ahead electricity market price during time step i , expressed
 220 in €/kWh_{el}, Δt is the time step between two points in time in the discretised OCP with a total of N
 221 time steps. \dot{Q}_b and \dot{Q}_p are the heat delivered by the base and peak plant unit, respectively. Similarly,
 222 p_b and p_p are the prices of the heat generated by both units. They are expressed in €/kWh_{th} and are
 223 constant in time. This price already includes the plant energy efficiency. In this study, only the ratio
 224 between the two prices is imposed, equal to $p_p/p_b = 2$ [40].

$$C_{HP} = \sum_{i=1}^N p_e(i) \dot{W}_i \Delta t \quad (2)$$

$$C_{PS} = \sum_{i=1}^N (p_b \dot{Q}_{b,i} + p_p \dot{Q}_{p,i}) \Delta t \quad (3)$$

225 The price of heat generation by the base unit is lower than that by a peak unit. Hence, the peak
 226 shaving objective causes a preference for the base unit and incentivises peak shaving. The heat pump
 227 objective incentivises heat generation on moments during which the electricity price is low. In this
 228 study, network flexibility is the only available tool in the OCP to create energy flexibility. By running
 229 the optimisation twice, once with network flexibility available, i.e. the supply temperature may change
 230 between its nominal value and a value that is 10 °C higher, and once with no network flexibility
 231 available, i.e. the supply temperature leaving the plant must remain equal to the nominal value, the
 232 network flexibility activation can be isolated. A more elaborate explanation on this workflow can be
 233 found in [41].

234 The OCP settings and models are elaborated on in Appendix A.

235 3.2. Heat demand profiles

236 Following the process in Figure 1 (steps 5-15), heat demand profiles containing building parameter
 237 uncertainties can be set up.

238 In a first part (steps 5-8), the heat demand profile for every neighbourhood in *GenkNET* is
 239 determined based on a minimum energy use optimisation. Starting from the geometries of the 7775
 240 buildings in Genk (step 1) and the neighbourhood construction year (step 3), building parameters are
 241 allocated to each building based on the TABULA archetype U-values [26]. Based on this data and the
 242 StROBe user behaviour profiles (step 4), van der Heijde calculated the building heat demand profiles
 243 [42], based on the 4th order TEASER RC-model [43]. For this he used the typical meteorological year
 244 of Uccle, Belgium. To calculate the heat demand profiles, van der Heijde made use of *modesto* to
 245 determine the heat demand profile that ensures thermal comfort with minimum energy use in every
 246 building. Finally, to reach one heat demand profile per aggregated *GenkNET* neighbourhood, the heat
 247 demand profiles of buildings belonging to one neighbourhood are summed.

248 In a second part (steps 9-13), the uncertainty on the heat demand profiles is calculated. To reduce
 249 the computational burden, the uncertainty in each neighbourhood is determined through the use of
 250 archetype buildings. The archetype building for a neighbourhood is characterised by the estimated
 251 average construction year of the neighbourhood (step 3) and the average building geometry. To obtain
 252 the average building geometry, the geometry of all buildings is required (step 2). The areas of the
 253 façades and roofs are merged towards 4 orientations (N, E, S, W), with a negligible loss of accuracy
 254 [44]. This simplifies calculating the average. This procedure is repeated for every neighbourhood and
 255 results in nine archetype buildings.

256 For these nine archetype buildings, distributions on the U-values of the roof, ground floor, exterior
 257 wall and windows are introduced along with variations on the window-to-wall ratio, based on the
 258 method of De Jaeger et al. [toekomstige paper Ina]. These distributions are estimated to be as realistic

259 as possible considering the scarcely available data of Genk [toekomstige paper Ina]. Note that building
 260 geometry is assumed to be known perfectly, as are user behaviour and weather predictions.

261 Using these distributions, 500 versions of every archetype building are generated. The
 262 distributions of all (9x500) generated building parameters can be seen in Figure 3. Next, yearlong
 263 simulations of the archetype buildings are carried out in Modelica using the IDEAS model library
 264 [45]. These simulations entail a 2-zone white-box model of the SH system consisting of ideal radiator
 265 heating. The user behaviour and weather as they were described in Section 2 are applied. This leads to
 266 the distribution in annual SH heat demand in *GenkNET* shown in Figure 4.

267 Based on the simulation results, load duration curves (LDC) of every variation are set up. The
 268 coefficient of variation² (CV) for one archetype was found to change in function of the expected SH heat
 269 demand of that building $\dot{Q}_{\text{arch,SH},\mu}$. Furthermore, the CV could be well estimated by an exponential
 270 in function of the expected SH heat demand of the archetype building, with a , b and c the fitting
 271 parameters that depend on the neighbourhood.

$$CV(\dot{Q}_{\text{arch,SH},\mu}) = a \exp(-b\dot{Q}_{\text{arch,SH},\mu}) + c \quad (4)$$

272 By stating that the archetype building heat demand is the average building heat demand in a
 273 neighbourhood with N_b buildings and expected heat demand $\dot{Q}_{\text{SH},\mu}$, the following expression for CV
 274 can be set up for each neighbourhood:

$$CV(\dot{Q}_{\text{SH},\mu}) = a \exp\left(-b \frac{\dot{Q}_{\text{SH},\mu}}{N_b}\right) + c \quad (5)$$

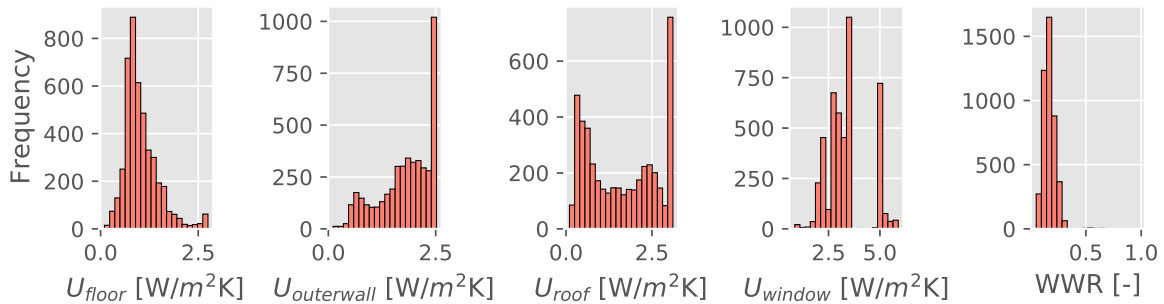


Figure 3. Five histograms showing the building parameter distribution in the nine *GenkNET* neighbourhoods, according to the 9x500 variations.

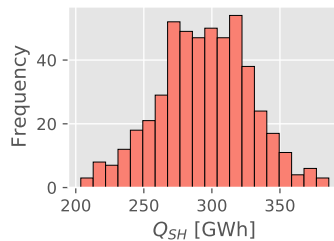


Figure 4. A histogram showing the distribution of the annual *GenkNET* heat demand, according to the 500 variations.

275 In a third part (steps 14-15), the uncertainties (steps 9-13) can be added to the optimal heat demand
 276 profiles (steps 5-8). Based on the exponential curves describing the CV, new SH heat demand profiles

² The coefficient of variation is the ratio of the standard deviation to the mean of a distribution: σ/μ .

277 for each neighbourhood in *GenkNET* are set up. To do so, a normal distribution is assumed, which
 278 has been used in the literature before to describe building heat or electricity demand distributions
 279 [23,29,32,38,46]. Considering the distribution in Figure 4, this seems a reasonable assumption. This
 280 allows the use of the following quantile function:

$$F^{-1}(p) = \mu + \sigma\sqrt{2}\operatorname{erf}^{-1}(2p - 1) \quad (6)$$

281 For a normally distributed variable, $F^{-1}(p)$ is the value of the variable for which there is a
 282 probability p such that $F^{-1}(p)$ is greater than or equal to the variable. In this equation, μ and σ are the
 283 expected value and standard deviation of the variable, and erf^{-1} is the inverse error function.

284 To set up the SH heat demand profile of version v , the following is done. The value for p is
 285 randomly selected from a uniform distribution between 0 and 1. Then, starting from the optimal SH
 286 heat demand profiles (steps 5-8) [42], at every point in time the quantile function is applied along with
 287 the CV that corresponds to the expected heat demand $\dot{Q}_{SH,\mu,i}$ at the point in time i :

$$\dot{Q}_{SH,v,i}(p) = \dot{Q}_{SH,\mu,i} \left(1 + \operatorname{CV}(\dot{Q}_{SH,\mu,i})\sqrt{2}\operatorname{erf}^{-1}(2p - 1) \right) \quad (7)$$

288 This process yields curves that are scaled by a factor changing through time, with the factor
 289 depending on the heat demand at that time.

290 An additional step is added to introduce a small amount of random behaviour. Following the
 291 autoregressive process AR(1), an extra term was added to the heat demand profile. This term has an
 292 autocorrelation of 0.75 between two subsequent points in time separated by 15 minutes and it has a
 293 standard deviation of 3% of $\dot{Q}_{SH,\mu,i}$, following the prediction error analysis in [47].

294 This way, 100 different SH heat demand profiles are generated for every neighbourhood. To end
 295 up with 100 different versions of *GenkNET*, one generated profile of every neighbourhood is grouped
 296 together. This grouping was done fully at random, although it could be argued that there might be
 297 correlations between neighbourhoods, e.g. if the U-values were underestimated in one neighbourhood,
 298 chances are that this happened in other neighbourhoods as well. However, this effect is not included
 299 here.

300 No uncertainty was added to the DHW heat demand, so these are simply added to the different
 301 SH heat demand profiles. Finally, the left of Figure 5 shows the heat demand of *GenkNET* of all 100
 302 versions for the *Trans_Negp_e* day. The right graph shows 11 selected profiles, spread over the entire
 303 range. Note that the range in variation is similar to that shown in Figure 4, with about a factor 2
 304 between the most extreme cases. The extra random changes that were added to the profile have little
 305 effect and do not change the overall behaviour.

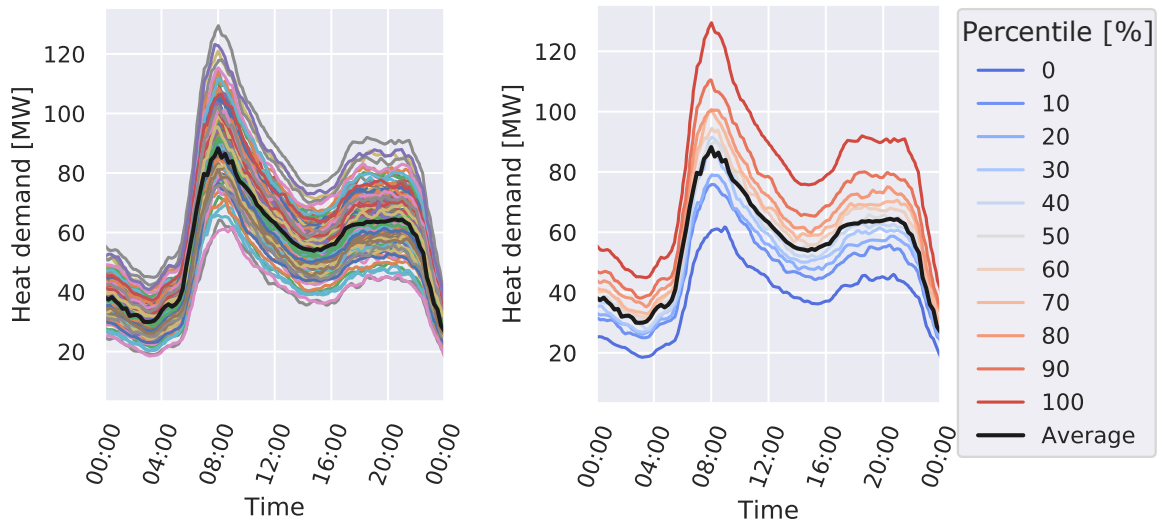
306 3.3. Uncertainty analysis

307 To analyse the influence of heat demand magnitude uncertainty on network flexibility activations,
 308 three steps are taken, which are described below.

309 3.3.1. Step 1: Optimal control of each *GenkNET* version

310 With 100 possible versions of *GenkNET* created, the optimal control strategy of every version can
 311 be calculated. By solving the OCP twice, once with and once without network flexibility, referred to as
 312 the Flexibility and Reference cases, the optimal network flexibility activation can be isolated. Six days
 313 (see Table 1) will be analysed with respect to operational heat pump optimisation, and peak shaving
 314 for different base load ratios (60, 80 and 95 %).

315 This leads to 100 optimal network flexibility activations for *GenkNET*, based on heat demand
 316 profiles that differ mostly in amplitude. This step will show how the optimal control changes as the
 317 heat demand magnitude changes.



(a) All 100 versions.

(b) 11 evenly spread out versions.

Figure 5. The different heat demand profiles of *GenkNET* for the *Trans_Negpe* day. The black line indicates the expected heat demand.

3.3.2. Step 2: Selection of 11 control strategies

All *GenkNET* versions are ordered from low to high annual heat demand. Using this ordering, every tenth profile is selected, corresponding to the selection in Figure 5b. Hence, when ordered according to the annual heat demand, versions 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 are chosen. These versions will be named by these numbers in the remainder of this study, with 0 corresponding to the *GenkNET* version with the lowest annual heat demand and 100 to the one with the highest annual heat demand. Note that the ordering of the profiles is the same for each of the six days and any other variation that is analysed in this study.

3.3.3. Step 3: Applying the 11 control strategies to all 100 *GenkNET* versions

Finally, the optimal control strategies of the 11 versions selected in the previous step are applied to all 100 *GenkNET* versions. This leads to 1100 evaluations of *GenkNET* for one day and one optimisation case. This step shows how the optimal control performance changes when the 'predicted' and 'actual' heat demand differ from each other.

4. Results

The results are split up into a discussion of the optimisation of heat pump operation and peak shaving optimisation and are presented below.

4.1. Operational heat pump optimisation

The operation of the heat pump is optimised to achieve the lowest possible electricity costs to drive the heat pump while delivering the heat demand to the customers. The electricity prices are based on the 2014 BELPEX day-ahead electricity market.

4.1.1. Optimal control of the 11 selected versions

In Step 1 of the uncertainty analysis, the optimal control strategies of all 100 versions of *GenkNET* were calculated. Figure 6 shows the optimal control of the 11 selected versions on the *Winter_Negpe* day (a winter day with an electricity price that becomes negative). These 11 versions are spread out over the entire range of heat demand magnitudes and give a good overview of the optimal control

343 of all versions. Figure 6 shows that the network is charged three times: during the two negative
 344 price periods and before a large change in electricity price. With the COP reducing when the supply
 345 temperature is increased, which is inevitable when activating network flexibility, these are the only
 346 moments when network flexibility is profitable. In Figure 6, the Flexibility and Reference case refer to
 347 the cases in which network flexibility is available and in which it is unavailable, respectively.

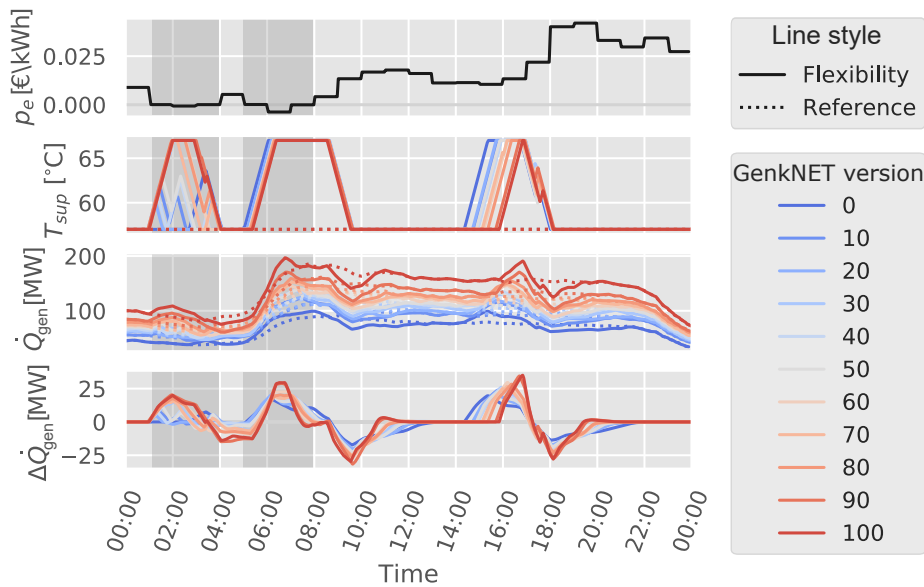


Figure 6. For the *Winter_Negp_e* day, the results of the operational heat pump optimisation for the 11 selected *GenkNET* versions are shown. From top to bottom, the electricity price, the supply temperature at the plant, the heat injection and the heat injection response (the difference between the Flexibility and Reference case) are shown. The negative price periods are indicated by the dark grey zones.

348 During the first negative price period, there is a substantial difference between the supply
 349 temperature pulses of the different *GenkNET* versions, whereas the pulses are nearly identical in the
 350 second negative price period. The differences in the first period are likely caused by the second period
 351 that follows shortly after. In the low heat demand versions, the water travels so slowly that charging
 352 the network during the first negative period causes the network to discharge during the second, more
 353 interesting, negative period, causing a loss in profits. In the high heat demand versions, the water
 354 travels faster and the discharge has ended by the time the second negative period starts.

355 When there is a large price difference, the supply temperature pulse remains similar in all cases
 356 but starts earlier as the heat demand reduces, again a consequence of the lower water speeds in the
 357 network. Hence, the general actions are largely based on the electricity price profile and remain
 358 similar throughout all versions. However, the exact timing can change considerably, with pulse lengths
 359 doubling as the heat demand becomes lower. Note that the third pulse always ends at the same point
 360 in time, namely when the price increase is taking place.

361 4.1.2. Applying 11 different control strategies to all 100 *GenkNET* versions

362 In Step 3, the optimal supply temperature profiles in Figure 6 are applied to all 100 versions. This
 363 leads to 1100 evaluations for each day, which are presented in Figure 7. Only the three days during
 364 which there is a significant network flexibility activation are shown: *Winter_Negp_e*, *Trans_Negp_e* and
 365 *Trans_LargeΔp_e*. For each of the 11 selected control strategies, a box plot is set up. The box plot presents
 366 the profit of applying the selected control strategy to all *GenkNET* versions. Studying the median value,
 367 it seems that every optimal control strategy achieves a similar profit on average. On the *Trans_Negp_e*
 368 and *Trans_LargeΔp_e* days, the spread on the profits remains similar as well, regardless of the control

369 strategy. The profits are symmetrically spread around the median and show a possible deviation from
 370 the median profit of about 20 % and 33 % on $Trans_Negp_e$ and $Trans_Large\Delta p_e$, respectively.

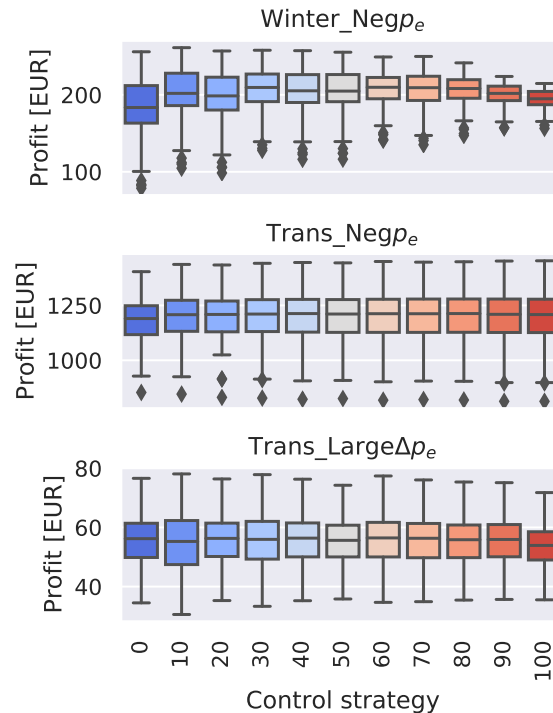


Figure 7. Box plots of the profits obtained with the 11 selected control strategies on all *GenkNET* versions for three different days. The box plots shows the median, first and third quartile and the minimum and maximum (excluding outliers) of a data set. The outliers are represented by the diamond markers.

371 On the $Winter_Negp_e$ day, the profit variation decreases as higher optimal control strategies are
 372 applied. Looking back at Figure 6, this is likely caused by the quick succession of two negative price
 373 periods. When a higher heat demand is predicted, a large supply temperature pulse is applied during
 374 the first negative period. If the actual heat demand is lower, the discharge phase is taking place during
 375 the more interesting second negative price period, limiting the profits. Vice versa, if a low heat demand
 376 is predicted, but it turns out to be high, the first negative price period was only covered by a small
 377 temperature pulse. The spread in profits for control strategies 0 and 100 can be seen in Figure 8, which
 378 also shows the difference with the actual optimal solution. It seems that an optimal control strategy
 379 based on a different heat demand prediction can lead to a profit reduction by up to a factor 2.

380 From the cases studied here, it seems that the risk related to heat demand magnitude uncertainty
 381 can cause a reduction in profits, yet there was no risk of losing money (negative profits). In general,
 382 the control strategy remained similar in all cases, as the control strategy mostly aims for moments with
 383 a negative price or with large price changes. The heat demand at those times seems less important.

384 4.2. Peak shaving optimisation

385 In the peak shaving optimisation, two plants are available to generate the heat. The base unit can
 386 generate heat cheaply but does not have a heat output sufficiently large to deliver the heat demand
 387 peaks. The peak unit can cover the peak but at a higher cost. To minimise the cost of heat generation,
 388 peak shaving is hence applied by activating network flexibility.

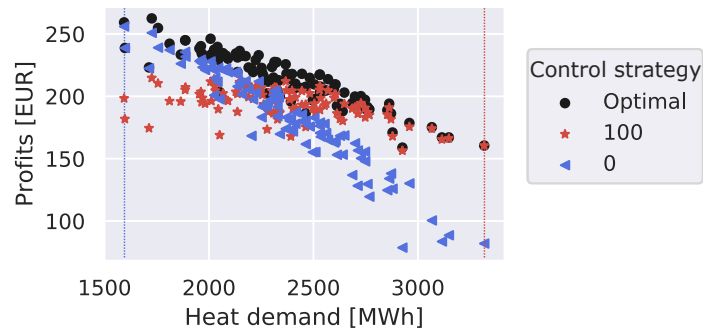


Figure 8. On the x -axis the heat demand during the *Winter_Negp_e* day is shown, on the y -axis the profit that was obtained. Every dot represents one version of *GenkNET* managed by one control strategy, the colour and marker shape indicate the applied control strategy. One version will always have the same heat demand, regardless of the applied control strategy. The vertical dotted lines show the heat demand that correspond to the 0 and 100 control strategies.

389 4.2.1. Optimal control of the 11 selected versions

390 Figure 9 shows the optimal control results of the 11 selected profiles when the base unit can deliver
 391 95 % of the expected peak heat demand, with a peak-base price ratio of 2 on the *Winter_Large Δp_e* day.
 392 It shows that the versions with a lower heat demand do not require any network flexibility, while those
 393 with a higher heat demand do not succeed in shaving the entire peak. For the versions with the highest
 394 heat demand, an additional large supply temperature pulse appears at the end of the peak period. As
 395 was explained in [6], a flexibility activation takes place in several phases. First, the network is charged,
 396 then it is discharged and at the end a rebound takes place. The rebound compensates the part of the
 397 discharge that was not covered by the initial charge. In the last network flexibility activation at the end
 398 of the peak period in Figure 9, the initial charge and discharge are both covered by the peak unit, but
 399 the rebound is covered by the base unit, effectively moving a small amount of energy from the peak
 400 unit to the base unit.



Figure 9. For the *Winter_Large Δp_e* day, the results of the base-peak plant optimisation for the 11 selected *GenkNET* versions with a base load ratio of 95 % are shown. From top to bottom, the supply temperature at the plant, the heat injection and the heat injection response are shown. In the middle graph, the maximum heat output of the base unit is indicated by the grey horizontal line.

4.01 The differences between the different control strategies are clearly larger than for the operational
 4.02 heat pump optimisation. Hence, it is expected that larger ranges of profits (and losses) will appear
 4.03 when applying these strategies to all 100 versions.

4.04 4.2.2. Applying 11 different control strategies to all 100 *GenkNET* versions

4.05 Figure 10 shows the peak energy that could be avoided in all 100 versions with 11 different control
 4.06 strategies. This is done for the *Winter_Large Δp_e* and *Trans_Small Δp_e* days for base load ratios of 60, 80
 4.07 and 95 %. The variation in avoided peak energy has a much larger range than the profit range found
 4.08 for the operational heat pump optimisation. When comparing base load ratios different trends can be
 4.09 observed.

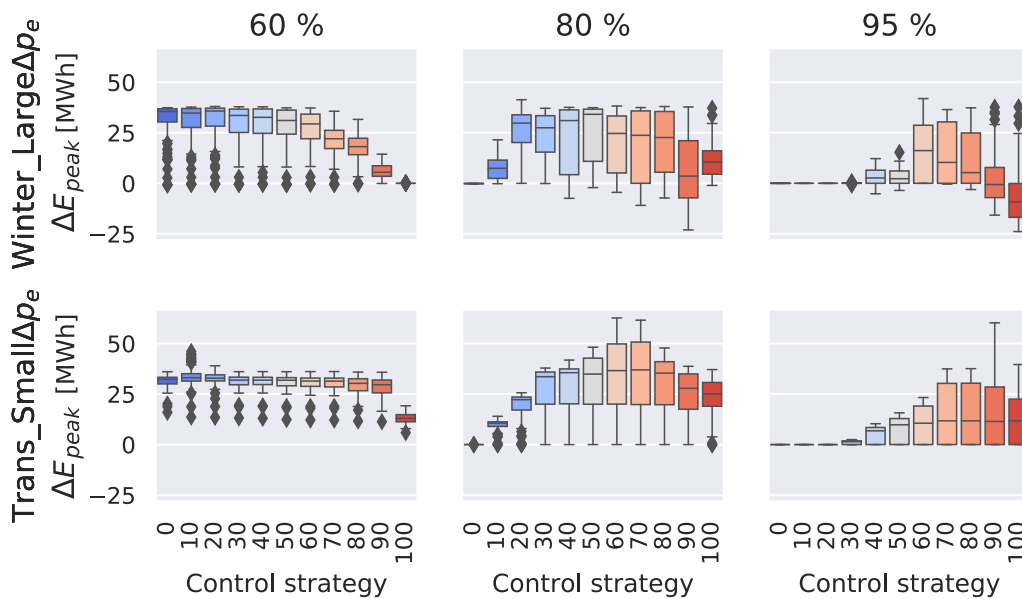


Figure 10. Box plots of the avoided peak energy with the 11 selected control strategies on all 100 versions of *GenkNET* for *Winter_Large Δp_e* and *Trans_Small Δp_e* and for different base load ratios.

4.10 Starting on the right of Figure 10 with a base load ratio of 95 %, the lower control strategies cannot
 4.11 accomplish anything at all. Looking back at Figure 9, no network flexibility is required when the heat
 4.12 demand is low, hence there is no network flexibility activation. Going to the higher control strategies,
 4.13 the average peak energy that can be avoided increases as does the possible range, although it always
 4.14 remains mostly positive, i.e. very little to no extra peak energy had to be generated even in the worst
 4.15 case for *Trans_Small Δp_e* .

4.16 However, the highest control strategies on *Winter_Large Δp_e* cause extra amounts of peak energy
 4.17 to be generated in many cases. Here, a second large temperature pulse at the end of the peak period
 4.18 has appeared (see Figure 9). This type of network flexibility appears to entail a large risk, as illustrated
 4.19 in Figure 11. If this second pulse is applied to a case with a lower heat demand in which the peak unit
 4.20 is not active at that time, the peak unit might have to be (re)activated to deliver this pulse while the
 4.21 base load is later on reduced, e.g. in version 50. This increases the delivered peak energy substantially.
 4.22 It seems that in case of a large base unit and heat demand magnitude uncertainties, it is better to
 4.23 overestimate than underestimate the heat demand, but to avoid a network flexibility activation at the
 4.24 end of a peak period.

4.25 Going to a base load ratio of 80 %, a similar pattern appears, yet everything has shifted to the left;
 4.26 the base unit must now be activated more quickly. The *Winter_Large Δp_e* day again shows a risk to
 4.27 generate more peak energy when going to higher control strategies. Again, this is caused by a network
 4.28 flexibility activation at the end of a peak period. This extra pulse has disappeared again at the highest
 4.29 control strategy, which shows no risk to increase the peak energy. The 80 % case shows a clear best

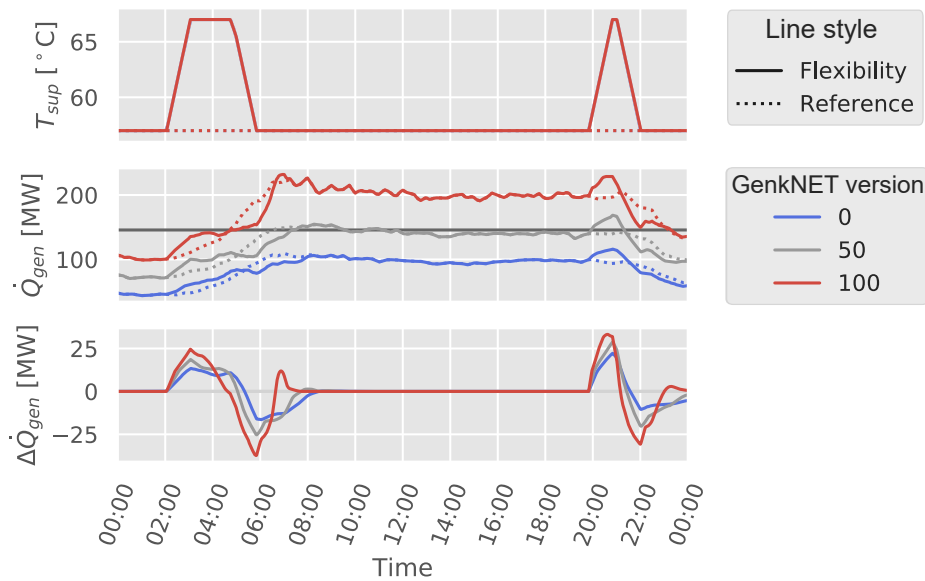


Figure 11. The application of control strategy 100 on a *GenkNET* version with low (0), medium (50) and high (100) heat demand on the *Winter_Large Δp_e* day with a base load ratio of 95 %. The maximum base unit heat generation is indicated by the horizontal grey line.

430 result in the intermediate control strategies. For these strategies, the heat demand was high enough
 431 that peak shaving is required, but not that high that the peak unit must be active (nearly) all the time,
 432 limiting chances for network flexibility.

433 In case of 60 %, another shift to the left has occurred, the peak unit is now activated even in case
 434 of the smallest heat demand. The average avoided peak energy now remains very constant up to the
 435 highest control strategies. Here, it decreases again, as hardly any network flexibility is activated any
 436 more. The heat demand has now become so high that the base unit must be active (nearly) all the
 437 time. Although the range of possible peak energy avoided can be large, there is little risk, i.e. the
 438 delivered peak energy will not increase. It seems that in case of a smaller base unit, it would be safe to
 439 underestimate the heat demand when deciding a control strategy.

440 To better understand what occurs when the heat demand changes and what influence the different
 441 control strategies have, Figure 12 shows the peak energy that could be avoided for all *GenkNET* cases
 442 in six days for a base load ratio of 95 %. For each *GenkNET* version, the result of four different control
 443 strategies is shown: the optimal control strategy of that version, the result when the lowest and highest
 444 heat demand are ‘predicted’ and lastly a control strategy that follows the recommendations from
 445 before. For a base load ratio of 95 %, it seemed advisable to select a control strategy corresponding to a
 446 higher heat demand, without going too high. Hence, control strategy 70 is selected.

447 The optimal control solutions (black dots) in Figure 12 show some interesting results. With low
 448 heat demands, there is no need for peak shaving and hence no peak energy can be avoided either.
 449 After a while, the avoided peak starts increasing, until on most days a maximum is reached. This
 450 maximum corresponds to the maximum energy storage capacity of the *GenkNET* network (estimated
 451 to be 36.8 MWh)³. When this maximum is exceeded, there are multiple peaks and network flexibility
 452 activations during one day, e.g. a morning and evening peak. These extra peaks appear and disappear
 453 as the heat demand magnitude changes.

³ This number was estimated by calculating the total water mass in the *GenkNET* network, see Figure 2, and multiplying this mass with the specific heat capacity of water and the allowed temperature increase of 10 °C.

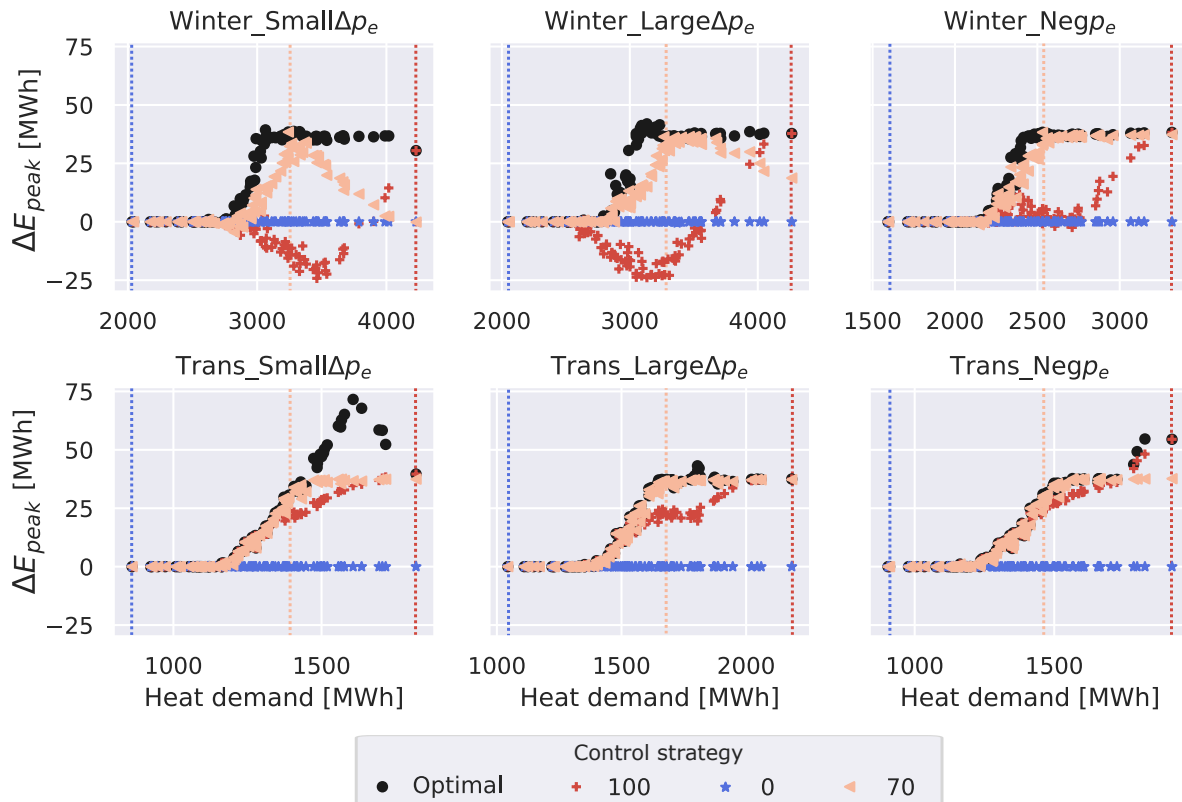


Figure 12. On the x-axis the total heat demand during each day and each version is shown, on the y-axis the peak energy that was avoided with a control strategy. Every dot represents one version of *GenkNET* managed by a different control strategy. The vertical dotted lines show the heat demand that correspond to the shown control strategies.

454 Of all control strategies, the optimal control strategy (black dots) reaches the best result in all
 455 cases, as is expected. Control strategy 0 cannot avoid any peak energy, as it does not activate network
 456 flexibility. Control strategy 100 can accomplish a reasonable result on the transitional days, noticing
 457 the afternoon peak that occurs with the highest heat demands on *Trans_Negp_e*. However, on the winter
 458 days, it attempts to activate network flexibility at the end of a peak unit activation, which is risky in
 459 case the heat demand turns out to be lower. Lastly, when control strategy 70 is applied, it can in most
 460 cases follow the optimal strategy very well. It only misses the afternoon peak of *Trans_SmallΔp_e* and
 461 *Trans_Negp_e*. On *Winter_SmallΔp_e*, its performance decreases with increasing heat demand. Here, the
 462 peak period moves significantly through time as the heat demand changes. In case of the highest heat
 463 demands, the peak has already started by the time case 70 starts charging the network. This implies
 464 that changes in peak timing will complicate the selection of a robust control strategy even further.

465 Remember that each *GenkNET* version is composed of nine neighbourhoods, of which the heat
 466 demand variation was chosen at random and independently of the other neighbourhoods. This means
 467 that two *GenkNET* versions with equal total daily heat demand, could have a different distribution
 468 of heat demand throughout the nine neighbourhoods and a different reaction to the same supply
 469 temperature pulse. Yet, in Figure 12, all points show clear trends. It seems that a different heat
 470 demand distribution amongst the *GenkNET* neighbourhoods does not influence the network flexibility
 471 activation that much with the currently imposed building parameter distributions.

472 5. Discussion

473 This study investigates how optimal control changes when the building parameters and hence the
 474 heat demand magnitude changes and how the control performance changes when a control strategy
 475 based on a different ‘predicted’ heat demand is applied.

476 *5.1. How does the optimal network flexibility activation change when the building parameters/heat demand*
477 *magnitude changes?*

478 When the heat demand changes so do the mass flow rates and the network flexibility timing. This
479 is an effect that is visible in the operational heat pump optimisations, although its influence is limited
480 when electricity prices become negative. Only in certain circumstances when multiple negative price
481 periods follow each other shortly, the heat demand magnitude may influence the network flexibility
482 activation to a larger extent.

483 In case of peak shaving, not only the mass flow rates in the system are important, so is the
484 magnitude of the heat demand with respect to the maximum heat output of the base unit. This can
485 largely influence the network flexibility activation. With a low heat demand, it may be that there is no
486 need for peak shaving, while with a high heat demand network flexibility is not sufficient to shave the
487 entire peak.

488 *5.2. How does the network flexibility performance change when the control strategy changes?*

489 There was relatively little difference in the performance of different control strategies for
490 operational heat pump optimisation. The resulting (average) profits were rather independent of the
491 applied control strategy. Again, in the considered cases it seems that the electricity price timing is at
492 least as important as the heat demand magnitude. Only in special cases (with multiple negative price
493 periods), a significant difference in control strategy performance could be noticed. Hence, for the case
494 of *GenkNET* with variations on building parameters, operational heat pump optimisation does not
495 present much risk. A minimum profit could be guaranteed in all cases.

496 For peak shaving, another observation can be made. The control strategies were highly dependent
497 on the heat demand magnitude. Yet, an analysis of their performance (i.e. the peak energy that could
498 be shaved) showed that the studied heat demand uncertainty does not introduce much risk. With most
499 control strategies, the generated peak energy remained the same or decreased. In the few cases that
500 network flexibility accomplished a result worse than the Reference case, this was caused by a network
501 flexibility activation at the end of a peak period, or by a large change in the start time of a peak period.
502 This does suggest that uncertainties in timing (related to user behaviour and weather), may induce
503 larger risks.

504 *5.3. Does this preliminary study lead to insights for a more robust activation of network flexibility?*

505 For operational heat pump optimisation, it seems electricity price related uncertainties may be
506 more relevant. Future research should look into this type of uncertainty in more detail to conclude what
507 measures are required to achieve a more robust network flexibility activation. For now, considering
508 only building parameter uncertainties, there seems to be little risk in selecting a control strategy.

509 For peak shaving, if only heat demand magnitude uncertainties are expected, a recommendation
510 for peak shaving can be made based on the results gathered in this paper. The losses associated with
511 activating network flexibility needlessly are limited, while the possible gains are substantial. Hence, it
512 seems better to activate too much flexibility, instead of too little. Only the activation at the end of a
513 peak period should be avoided as it introduces a risk of generating more peak energy.

514 *5.4. Remarks*

515 This study only considers uncertainties on building parameters. Yet, in reality, these uncertainties
516 may be the least problematic with respect to control. As building parameters do not change, the
517 errors in heat demand predictions caused by them can be corrected over time. By contrast, the user
518 behaviour and weather do change over time and may be much harder to deal with. However, for
519 building parameter data, realistic data distributions could be set up and analysed.

520 The plant models are simple and do not contain all relevant aspects. For example, the peak
521 unit may require start-up and shut-down costs. These costs increase the risk associated with heat

522 demand uncertainties, as a small peak unit activation may cost much more than was assumed now.
523 Additionally, it was assumed that the base unit can increase the network supply temperature without a
524 reduced efficiency. If the efficiency depends on the supply temperature, this may alter the conclusions
525 made before. Similar things can be said for the heat pump, for ramping limits and costs, etc. A future
526 study should look into these aspects.

527 The building heat demand profiles were built with small random variations, along with
528 uncorrelated building parameter uncertainties between the neighbourhoods. The results suggest
529 that these aspects have little influence, as the scatter plots of Figures 8 and 12 showed clear trends
530 when ordering the different versions according to the total heat demand during the day. This would
531 indicate that 1) small (random) heat demand variations may not be that important. Hence, predictions
532 may not need to go in great detail, although the extent of this should be investigated. 2) The distribution
533 of the heat demand among the neighbourhoods may not have such a large influence either, although
534 the different neighbourhood locations in the network do influence the network flexibility activation
535 timing. Again, this is another aspect that merits further study.

536 6. Conclusion

537 This study evaluates the influence of building parameter uncertainties on network flexibility
538 performance. This is done by determining and analysing distributions for building parameters in the
539 city of Genk, Belgium. This led to 100 different profiles describing the heat demand in a fictive DH
540 system in Genk. These heat demand profiles differ mostly in magnitude, not in timing. The optimal
541 control strategy applying network flexibility for these different heat demand profiles was calculated
542 for operational heat pump optimisation and peak shaving. Additionally, control strategies that are
543 optimal for one heat demand profile were applied to all others, to study the influence of an incorrect
544 heat demand prediction.

545 Analysis of these results shows that building parameter uncertainties do not influence operational
546 heat pump optimisation much, and could reach an average profit that is similar regardless of the
547 applied control strategy. For peak shaving, the heat demand magnitude matters much more, as it is
548 the main factor that determines a peak unit activation. Yet, here the risk remains limited, hence a large
549 network flexibility activation to prevent a possible peak period seems advisable.

550 **Author Contributions:** Conceptualization, Annelies Vandermeulen, Ina De Jaeger, Tijs Van Oevelen, Dirk Saelens
551 and Lieve Helsen; Data curation, Ina De Jaeger; Formal analysis, Annelies Vandermeulen; Funding acquisition,
552 Annelies Vandermeulen, Ina De Jaeger, Dirk Saelens and Lieve Helsen; Investigation, Annelies Vandermeulen and
553 Ina De Jaeger; Methodology, Annelies Vandermeulen and Ina De Jaeger; Project administration, Dirk Saelens and
554 Lieve Helsen; Software, Annelies Vandermeulen and Ina De Jaeger; Supervision, Tijs Van Oevelen, Dirk Saelens
555 and Lieve Helsen; Validation, Annelies Vandermeulen and Ina De Jaeger; Visualization, Annelies Vandermeulen
556 and Ina De Jaeger; Writing – original draft, Annelies Vandermeulen and Ina De Jaeger; Writing – review & editing,
557 Annelies Vandermeulen, Ina De Jaeger, Tijs Van Oevelen, Dirk Saelens and Lieve Helsen.

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564 **Conflicts of Interest:** The authors declare no conflict of interest.

565 Abbreviations

566 The following abbreviations are used in this manuscript:

567

CHP	Combined Heat and Power
CV	Coefficient of Variation
DH	District Heating
DHC	District Heating and Cooling
DHW	Domestic Hot Water
568 LDC	Load Duration Curve
MPC	Model Predictive Control
OCP	Optimal Control Problem
R ² ES	Renewable and Residual Energy Sources
SH	Space Heating
TES	Thermal Energy Storage

569 Appendix A. Optimal control component models and settings

570 This appendix shortly presents the component models included in the OCP solved in this paper,
571 along with the optimisation settings. For a more detailed overview of the applied optimal control
572 problem, please refer to [48].

573 Appendix A.1. Pipe model

574 The pipe model is an explicit transient first-order upwind finite volume model, as has been used
575 before in the literature [49–56]. The energy balance of one finite volume is described in Equation A1.
576 m_k is the mass of water in one finite volume, c_p is the specific heat capacity of water, i and k are indices
577 indicating the time step and the finite volume, respectively. $T_{i,k}$ is the temperature of one finite volume
578 at one instance in time, Δt is the length of the time step, \dot{m} is the mass flow rate through the pipe, T_g is
579 the ground temperature and R is the thermal resistance between water and surrounding ground.

$$m_k c_p (T_{i,k} - T_{i-1,k}) + \dot{m}_i c_p \Delta t (T_{i-1,k} - T_{i-1,k-1}) = \frac{T_g - T_{i-1,k}}{R} \Delta t \quad (\text{A1})$$

580 To account for the wall thermal inertia, a correction at the end of a pipe has been added. This
581 correction is given in Equation A2 and follows the technique presented by Benonysson [57]. Here,
582 $T'_{\text{out},i}$ is the temperature exiting the pipe corrected for the wall thermal inertia, while $T_{\text{out},i}$ is the
583 temperature exiting the pipe as calculated by the finite volume model. C_{pipe} is the thermal capacity of
584 of the pipe wall, $T_{\text{wall},k}$ is the pipe wall temperature. The wall temperature is updated through time
585 by Equation A3

$$T'_{\text{out},i} = \frac{T_{\text{out},i} \dot{m}_i c_p \Delta t + C_{\text{pipe}} T_{\text{wall},i-1}}{C_{\text{pipe}} + \dot{m}_i c_p \Delta t} \quad (\text{A2})$$

$$T_{\text{wall},i} = T'_{\text{out},i} \quad (\text{A3})$$

This finite volume model is only stable if the following condition related to the spatial and temporal discretisation is met:

$$\text{CFL} = \frac{u \Delta t}{\Delta x} \leq 1 \quad (\text{A4})$$

586 In this equation, u is the speed of water through the pipe and Δx is axial length of a finite volume.
587 The closer CFL is to one, the less numerical diffusion takes place and the more accurate the model is.
588 Similarly, the finer the discretisation is, i.e. the smaller Δx and Δt are, the more accurate the model is.
589 However, a finer discretisation causes a quadratic increase in calculation time. To discretise, a careful
590 selection of the spatial discretisation (Δx) and temporal discretisation (Δt) were made such that the
591 accuracy is sufficiently high and the problem remains solvable within an acceptable time. The model
592 accuracy was tested by comparing it to Modelica simulations of both a validated pipe model [58] and

593 a *GenkNET* DH system model consisting of detailed component models. For more information, please
594 refer to [48].

595 *Appendix A.2. Substation model*

596 The substation model that is included in the OCP is the *No HEx* model derived and described
597 in [6]. This model is based on a substation with two heat exchangers, one for space heating (with
598 radiator heating) and one for domestic hot water (DHW). Although the building heating system and
599 the heat exchangers are modelled, the heat demand profiles are determined in advance [42], hence
600 no building structure model is included. The *No HEx* model is a simplified version of the original
601 detailed substation model. *No HEx* only includes a 2D look-up table that gives the primary return
602 temperature exiting the substation in function of the space heating and DHW heat demand and the
603 incoming primary supply temperature.

604 Again, this model was tested and verified by comparing it with simulation of a *GenkNET* DH
605 system model consisting of detailed component models.

606 *Appendix A.3. Heat pump model*

607 The *GenkNET* central heat pump is an electric air-to-water heat pump. An important parameter is
608 the coefficient of performance (COP). This is the ratio of \dot{Q}_{gen} the heat supplied to the DH system to \dot{W}
609 the electrical power required.

$$\text{COP} = \frac{\dot{Q}_{\text{gen}}}{\dot{W}} \quad (\text{A5})$$

610 Representing a heat pump by a Carnot cycle, the COP can be expressed as a function of the
611 condenser and evaporator temperatures, corresponding to $T_{\text{gen, sup}}$ and T_e , the DH supply and ambient
612 air temperatures, respectively. As real heat pumps do not follow the ideal Carnot cycle, an additional
613 efficiency η_C is introduced, taking the value of 0.6 [42]. η_C incorporates the efficiency loss due
614 to non-adiabatic compression, isenthalpic expansion, non-isothermal heat exchange, etc. The air
615 temperature T_e corresponds to the typical meteorological year in Uccle (BE).

$$\text{COP} = \eta_C \frac{T_{\text{gen, sup}}}{T_{\text{gen, sup}} - T_e} \quad (\text{A6})$$

616 The plant then delivers heat to the DH system, according to Equation A7, with \dot{m}_{gen} , $T_{\text{gen, sup}}$ and
617 $T_{\text{gen, ret}}$ the DH mass flow rate, supply and return temperatures at the plant, respectively.

$$\dot{Q}_{\text{gen}} = \dot{m}_{\text{gen}} c_p (T_{\text{gen, sup}} - T_{\text{gen, ret}}) \quad (\text{A7})$$

The following constraint to limit temperature ramping is added:

$$\frac{-10^\circ\text{C}}{3600\text{ s}} \Delta t \leq T_{\text{gen, sup, i}} - T_{\text{gen, sup, i-1}} \leq \frac{-10^\circ\text{C}}{3600\text{ s}} \Delta t \quad (\text{A8})$$

618 This equation limits the supply temperature changes between two points in time (i and i-1),
619 separated by Δt seconds in accordance with EN 13941 [59]. Additionally, the supply temperature can
620 only change between the nominal value $T_{\text{sup, nom}}$ and a temperature that is 10 °C higher, giving it the
621 required degree of freedom to activate network flexibility:

$$T_{\text{sup, nom}} \leq T_{\text{gen, sup}} \leq T_{\text{sup, nom}} + 10^\circ\text{C} \quad (\text{A9})$$

622 The heat output is only constrained to be positive, as in Equation A10. There is no maximum
623 value the heat output can take, nor any limit on how fast the heat output can increase. However, with
624 the temperature ramping constraint in place and the *GenkNET* heat demand profiles determined in
625 advance, the values the plant heat output can take will at all times be acceptable.

$$0 \leq \dot{Q}_{\text{gen}} \quad (\text{A10})$$

626 Appendix A.4. Base and peak plant model

627 In a second possible heat generation site, a base and peak plant work together. The base plant is
 628 cheap, but has a maximum heat output that is insufficient to deliver all heat demand. The peak plant
 629 is more expensive, but can supplement the base heat output to deliver all heat demand. In this case
 630 peak shaving could reduce operational costs.

The base and peak plant are modelled as follows:

$$\dot{Q}_{\text{gen}} = \dot{Q}_{\text{b}} + \dot{Q}_{\text{p}} = m_{\text{gen}} c_p (T_{\text{gen, sup}} - T_{\text{gen, ret}}) \quad (\text{A11})$$

631 with \dot{Q}_{gen} the total heat generated by both base and peak plant and \dot{Q}_{b} and \dot{Q}_{p} is the heat generated
 632 by the base and peak unit separately.

Along with the constraints in Equations A8 and A9, the following limits on heat output are also included:

$$0 \leq \dot{Q}_{\text{b}} \leq \dot{Q}_{\text{b, max}} \quad (\text{A12})$$

$$0 \leq \dot{Q}_{\text{p}} \quad (\text{A13})$$

633 The base plant heat output is limited by $\dot{Q}_{\text{b, max}}$. The peak plant, just like the heat pump, does not have
 634 any limit on the maximum power output. Again, the heat output will be limited due to the pre-defined
 635 heat demand profiles and supply temperature constraints.

636 Appendix A.5. Optimal control model settings

637 The horizon of the optimisation problem is 24 hours, with a time step that changes through time,
 638 but is always smaller than 5 minutes. Each pipe in the network contains at least 3 and at most 22
 639 finite volumes. These measures keep the CFL-number as close to 1 as possible and the problem size as
 640 limited as possible. The combination of all component models previously presented with mass and
 641 energy balances, leads to a non-linear program that is solved in modesto [39] with ipopt [60].

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